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Fluctuations and finite-size effect in the Bak-Sneppen model

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Abstract. The self-organized criticality in the nearest-neighbor version of the Bak-Sneppen model is investigated from the event-by-event fluctuations of the mean fitness. The finite-size effect on the evolution of the critical state is shown, and a scaling solution to the gap equation for an infinite one-dimensional lattice is given numerically for the first time. The mean lifetime of avalanches is presented as a function of the gap from the solution. The critical value of the gap and an exponent are calculated from the solution.

PACS. 64.60.-i General studies of phase transitions – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 87.10.+e General theory and mathematical aspects

1 Introduction

The phenomenon of "self-organized criticality" (SOC) has become a topic of considerable interest in the investigation of complex systems. The interest originates from the novel laws governing the phenomenon and the potential applications which range from the behavior of a sandpile and the description of the growth of surfaces to a generic description of biological evolution [1–10]. It is observed that, contrary to our naive expectations, the dynamics of complex systems in nature does not follow a smooth and gradual path, instead it often occurs in punctuated bursts or "avalanches". The nonequilibrium systems can evolve towards a critical state with fractal properties in the space and time dimensions. No simple rule can be used to describe the spatial-temporal complexity. Power-law distributions for the spatial size and lifetime of the "avalanches" have been observed in various complex systems and are regarded as "fingerprints" for SOC. Such complexity also shows up in simple mathematical models for biological evolution and growth phenomena far from equilibrium. However, it is fair to say that our understanding of the nature of SOC is still in its infancy, since up to now no commonly accepted strict meaning of SOC exists even though a minimal definition was given in [11] and renewed in [12].

One of the simplest SOC models is the one-dimensional Bak-Sneppen (BS) model. There are so called nearest- and random-neighbor BS models which are introduced in [13–15] to mimic biological evolution. In this paper only the nearest neighbor BS model will be considered because that version is widely regarded as one of the simplest realizations of SOC. Unlike the random-neighbor version of the BS model [16], quantities in the nearest-neighbor version cannot be analytically calculated.

In the BS model random numbers (fitness) ξ_j ($j = 1, 2, \cdots, L$) uniformly drawn from (0, 1) are assigned in the initial state to the L sites of a one-dimensional lattice. In each update a site with minimum fitness is to be located, and that site and its two nearest neighbors are assigned new random numbers which are also drawn uniformly from (0, 1). The maximum of the minimum fitness before sth update is called the gap G at time s. With this update procedure the gap G increases and at last approaches a critical value f_c.

In the BS model an avalanche with a gap G is defined as the update process as long as the gap G remains unchanged. Therefore it begins when the gap G is reached for the first time and ends when a new, larger gap is obtained. The size (or lifetime) of an avalanche is the number of time steps involved in the avalanche. Denote $\langle S\rangle_G$ as the average size of avalanches with a fixed gap G. According to the update rules of the BS model the distribution of random numbers above the gap G on the sites is flat, so that the average jump size in the gap at the completion of each avalanche is $(1 - G)/L$. For any selected resolution $\Delta G \ll 1$ there is a system size L sufficiently large that many avalanches are needed to increase the gap from G to $G + \Delta G$. Therefore the average number of avalanches required to increase the gap by ΔG is $N_{\rm av} = \Delta G L/(1-G)$. When $L \gg \Delta G^{-1}$ we ensure that $N_{\rm av} \gg 1$. In the large L limit $N_{\rm av}$ can be arbitrary large, and with such a limit the average number of time steps required for the gap increase is given by the interval $\Delta s = \langle S\rangle_G N_{\rm av}$. From the law of large numbers the fluctuations of this interval relative to its average value vanish as $\Delta G \to 0$. Thus we get $\Delta G/\Delta s = (1 - G)/(L\langle S\rangle_G)$. By taking the continuum

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limit a gap equation was derived in [17] which in our case is

$$\frac{\mathrm{d}G}{\mathrm{d}s} = \frac{1-G}{L\langle S\rangle_G} \,. \tag{1}$$

The gap equation describes how the system is driven to its critical state and plays a fundamental role in understanding the nature of the BS model. Once the solution to the gap equation is known, the exponents associated with the critical state and the critical point f_c can be determined. Unfortunately, the solution has never been obtained analytically nor numerically in any previous investigations of the model. The main obstacles are that the gap $G(s)$ is a random number in an update process for finite L and that equation (1) is exact and meaningful only for $L \to \infty$. To the best of our knowledge, $\langle S\rangle_G$ is not known as a function of the gap G analytically or numerically, and only an ansatz on its behavior was given in [17] for G quite close to its critical value f_c. More fundamentally, we should ask whether the number of updates s in equation (1) is the best variable to describe a system approaching its critical state. We will show in this paper that the answer to this question is negative.

In the gap equation the mean avalanche lifetime $\langle S\rangle_G$ is normally an implicit function of both the gap G and lattice size L. In the evolution of an avalanche which lasts S updates, only a finite number (no more than $S+2$) of sites are involved. If the lattice size L is very large, the avalanche can hardly detect the size of the lattice. Thus $\langle S\rangle_G$ may have weak dependence on L and will approach a certain finite constant in the limit $L \to \infty$ for given $G < f_\mathrm{c}$. Then from equation (1) one can see that the gap G will be asymptotically a function of a scaled variable $t = s/L$. The scaled variable t has a simple physical meaning. In the one-dimensional BS model three sites are assigned new random numbers in each update. After s updates $3s$ random numbers are updated (some sites undergo the update more than once). Thus t is one third the average number of updates each site undergoes. Since the only way for the system to approach its critical state is the update of the random fitness on the sites involved, t is a more suitable variable than s to describe the evolution of the system to the critical state.

For finite L the gap G is random and depends on s and L separately, and the departure of $G(s,L)$ from its asymptotic scaling form $G(t)$ reflects the finite-size effect in the process to SOC. An interesting question has never been answered: What is the fluctuation property of $G(s,L)$ if the same evolution is repeated many times? Though $G(t)$ is essential and important theoretically because only $G(t)$ can show the true nature of the BS model without additional influence from the finiteness of the lattice used, one cannot get the scaling function $G(t)$ directly since in real investigations the lattice size is always finite and the consumption of computing time and capacity increases quickly with the lattice size L. So another question can be raised: Can we get the limiting behavior $G(t)$ from $G(s,L)$? If yes, how? We will give answers to all the questions in this paper.

In this paper, we try to investigate the gap G as a function of the scaled time t for $L \to \infty$ and $\langle S\rangle_G$ as a function of G. The investigation can check the validity of the ansatz used in [17] and give a complete picture on how fast the system is driven to its critical state. We will show in this paper that the ansatz is true only for the gap G very close to its critical value f_c. Differing from usual studies of the model, we focus our attention on the event-to-event fluctuations of the fitness and the finite-size effect on the evolution. We will show that only such a study enables one to extract the limiting scaling solution $G(t)$ to the gap equation.

This paper is arranged as follows. In Section 2 we investigate the fluctuation property of the BS model. From the fluctuations we can get the limiting scaling behaviors of interesting quantities in Section 3. The critical value of the gap and an exponent are also given in this section. Section 4 contains conclusions.

2 Fluctuations in the BS model

In [18] it is suggested to investigate the mean fitness of a lattice at the sth update in a simulation of the BS model. The average is over all the L sites. We denote an evolution process from the initial state by event i, and the mean fitness of the event for a system at update s is then

$$f_i(s,L) \equiv \bar{\xi}(s) = \frac{1}{L}\sum_{j=1}^{L} \xi_j(s) \,. \tag{2}$$

With $f_i(s,L)$ instead of the minimum of ξ_j a different hierarchy of avalanches can be found. Different from the minimum of the fitness ξ_j at time s this averaged fitness $f_i(s,L)$ is a global quantity. Since the lattice size L and various exponents for the model are global quantities, one has reason to expect that $f_i(s,L)$ is more suitable than the minimum of ξ_j for the study of global properties such as the finite-size effect. We will show below that interesting properties of the BS model can indeed be obtained from $f_i(s,L)$.

Suppose that we were able to perform exactly the same updating process on an infinitely large lattice. Because $\langle S\rangle_G$ is finite for $G < f_\mathrm{c}$, a higher gap can be expected after approximately $\langle S\rangle_G$ updates. Due to the infinity of the lattice size L, there are infinitely many sites with fitness in any finite interval ΔG. So, the increase of the gap can only be infinitesimal, and the scaled time spent for such an increase of the gap is also infinitesimal. Therefore, the gap equation has a continuous solution for $L \to \infty$. In an avalanche with gap $G < f_\mathrm{c}$, there would be only a finite number of sites with fitness smaller than G. After the average over all ξ_j on the infinite number of sites is done, the finite number of sites with fitness less than G would have zero contribution to f_i, and one would have

$$f_i = \frac{1+G}{2} \tag{3}$$

since an infinite amount of random numbers ξ_j should be distributed uniformly in the range of $(G,\ 1)$. No fluctuation on the mean fitness f_i could be observed for an infinite lattice.

Now we turn to a realistic finite lattice system. Suppose that the system has experienced s updates and obtained a gap G. Then there will be a nonzero fraction of sites with fitness less than G during an avalanche. For such a system $f_i(s, L)$ will be different from event to event due to the randomness of the fitness ξ_j assigned to each site. In other words, $f_i(s, L)$ is also a random number, and there exist fluctuations in $f_i(s, L)$ from one evolution event to another. Normally, the fluctuations will make it more difficult to investigate some interesting quantities. We will show, however, that the fluctuations in $f_i(s, L)$ can be studied in an event-by-event way and can give us some useful information about the essence of the model. The event-by-event analysis method has recently been used in experimental data analysis in high energy physics [19–25] as a powerful tool to study the nature of fluctuations of some global quantities. As stated above, the mean fitness of the system is a global variable, so the event-by-event analysis method may be useful in extracting from the fluctuations meaningful information on the evolution of the system.

Some features of the fluctuations of the averaged fitness $f_i(s, L)$ can be foreseen from the updating rules of the BS model. Since the periodic boundary conditions are adopted in the model, the L sites are equivalent, and all the random fitness ξ_i on the sites will, as a consequence of the equivalence of the sites, satisfy the same distribution, although it is nonuniform and still unknown. Then one can expect, for an evolution event i with lattice of size $L \gg 1$,

$$f_i(s, L) = \langle f \rangle + \frac{r_i}{\sqrt{L}} \tag{4}$$

according to the central-limit theorem. In the last equation $\langle f \rangle$ is the ensemble average of the random mean-fitness $f_i(s, L)$ over many evolution processes from the same initial state, and r_i is a random number satisfying Gaussian distribution with zero mean and a width of $O(1)$ and represents the fluctuations of $f_i(s, L)$ in an evolution at update s for a system with size L.

We now can probe into the properties of the fluctuations of $f_i(s, L)$. If there were, at update s, the same nonzero fraction of sites with fitness ξ_j less than the gap G in simulations with lattices of different size L, the random numbers ξ_i would be distributed uniformly in the regions $(G, 1)$ but non-uniformly in $(0, G)$ (distributions in both regions are determined by the gap and the fraction of sites with $\xi_j < G$), as can be seen from the randomness of the fitness ξ_j on each site and the update rules of the BS model. Then both $\langle f \rangle$ and r_i should be independent of L. The assumption of a fixed nonzero fraction of sites with ξ_j less than G would have two consequences: (1) $\langle f \rangle$ less than $(1 + G)/2$ by a constant for all L; (2) the product of the width σ of the fluctuations of f_i and the square root of L, denoted as $\sigma_L \equiv \sigma\sqrt{L}$, would also be a constant for all L. Here, σ_L is the width of the distribution of r_i. Of course, the assumption of fixed fraction of fitness over the gap G is not true in the real case. In real simulations of the model with finite L, the number of sites involved in an avalanche with gap G (only those sites can have fitness less than the gap) is almost independent of the lattice size as long as the system is large enough. This means that the fraction of sites with fitness less than G certainly depends on the lattice sites L. On average, the smaller the L, the larger the fraction. Such dependence of the fraction on L will result in the L dependence of the Gaussian distribution of $f_i(s, L)$: The smaller the lattice size L, the smaller the $\langle f \rangle$ while the width of r_i is larger. So both $\langle f \rangle$ and σ_L depend on L.

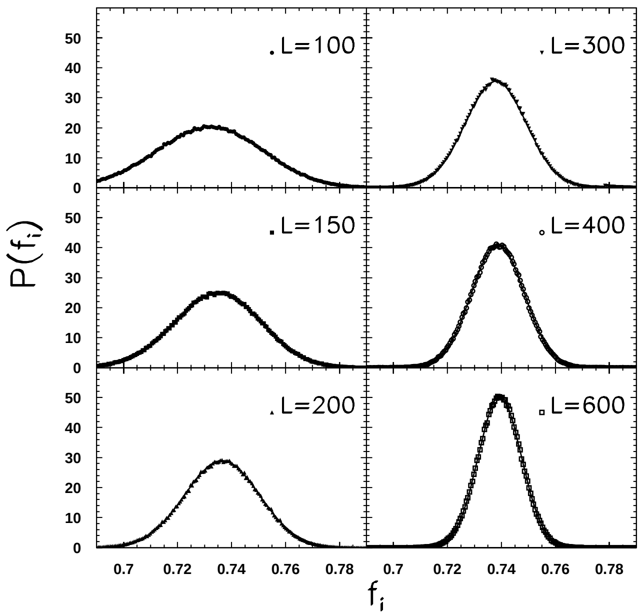

Fig. 1. Distributions of the average fitness $f_i(s, L)$ at a scaled time $t = s/L = 3.0$ for six lattice sizes L. Gaussian fitted curves are drawn in solid curves.

To verify our predictions, six simulations of the BS model with $L = 100$, 150, 200, 300, 400 and 600 are done. In each evolution of the simulation from the initial state 2000 updates are performed. Since we are interested in the fluctuations of f_i and the finite-size effect we first look at the distributions of $f_i(s, L)$ at a scaled time $t = s/L = 3.0$ for illustration. The normalized distributions for f_i are shown in Figure 1 for six L. All of them are of perfect Gaussian over three orders of magnitude. The distributions of f_i at other scaled time t are also Gaussian. This means that all the evolution processes have the same statistical property once they are viewed at the same scaled time t. This shows an advantage of variable t over the natural one s. From this figure one can get the peak positions $f_{\rm P}$ and the widths σ. $f_{\rm P}$ and σ_L are shown in Figure 2 as functions of $1/L$. As announced in the last paragraph, with the increase of L, $f_{\rm P}$ increases since the peaks of the Gaussian distributions in Figure 1 shift slightly towards the right but σ_L decreases slightly.

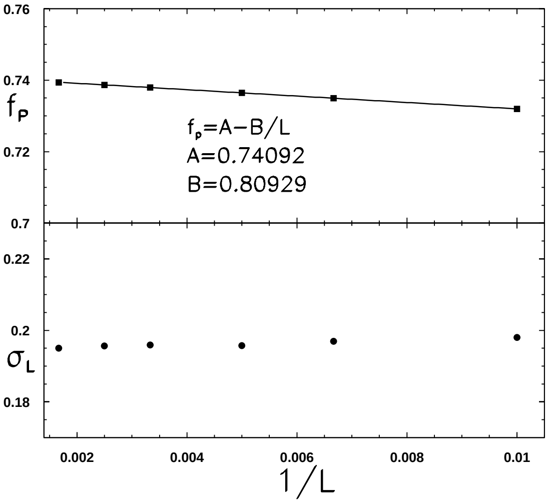

Fig. 2. The peak f_{P} and σ_L, the product of $\sqrt{L}$ and the width σ, of the Gaussian distributions in Figure 1 as functions of $1/L$ for $t = 3.0$.

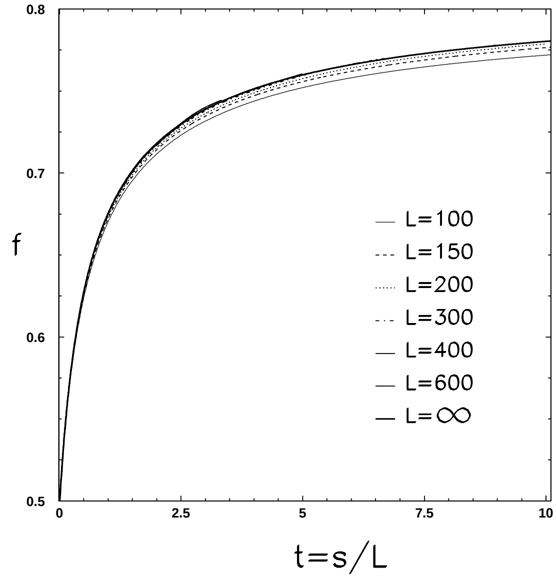

Fig. 3. Behaviors of the sample averaged fitness $f(t, L)$ as functions of t for six different lattice sizes L.

3 Size dependence and limiting behaviors in the BS model

Since the fluctuations in f_i are of Gaussian type, the mean $\langle f \rangle$ can be obtained without fitting the distribution of f_i if one additional average is made over a sample of N_{event} ($\gg 1$) evolution events from the initial state. Thus we have

$$f(t, L) = \frac{1}{N_{\mathrm{event}}} \sum_{i=1}^{N_{\mathrm{event}}} f_i(s, L) \simeq \langle f \rangle = f_{\mathrm{P}} \ . \quad (5)$$

Theoretically, there will still exist fluctuations in $f(t, L)$ as long as N_{event} is finite. From the central-limit theorem again the width of the reduced Gaussian fluctuations in $f(t, L)$ is now of the order of $1/\sqrt{L\,N_{\mathrm{event}}}$. Without any difficulty one can always choose N_{event} quite large, a few hundred thousand for example. With such a big event sample there is in fact no visual fluctuation in $f(t, L)$ which can be taken equal to $\langle f \rangle$ for any t and L.

To see the behavior of $f(t, L)$ and to study the finite size effect $f(t, L)$ is shown in Figure 3 as functions of t from the six simulations with different lattice size L mentioned above. In each simulation, N_{event} is chosen to be 500 000. From the curves in Figure 3 the finite-size effect can be clearly seen. It is shown that the difference between the curves with $L = 100$ and 150 is the largest for fixed t while that between $L = 400$ and 600 is very small. This means the curve with $L = 600$ is already quite close to the asymptotic one. One can see in detail the L dependence of $f(t, L)$ from the curves in Figure 3 by looking at the points corresponding to the same t for different L. For example, from the values of $f(t, L)$ at $t = 3.0$ which have already been given in upper part of Figure 2, the dependence of $f(t, L)$ on L can be extracted. It is fascinating to notice that the six points at $t = 3.0$ can be fitted well by the following expression

$$f(3, L) = f_{\mathrm{P}} = A - B/L \ , \quad (6)$$

with $A = 0.74092$ and $B = 0.80929$. This L dependence form of $f(t, L)$ can, of course, be anticipated. But one should notice that such a simplicity can be shown only in the study of fluctuations of the mean fitness with the scaled time t. Because of such a L dependence the horizontal axis in Figure 3 is chosen to be $1/L$ instead of L. The fitted curve is also shown in the upper part of Figure 2 as a solid curve. Since the six points can be well fitted by equation (6), one may wonder to which L the fitting formula is usable. One very special lattice with $L = 3$ should be mentioned here. On the one hand, for such a lattice all the three sites are always involved in *every* update, thus one cannot expect the occurrence of any critical state. On average, the state of the lattice is the same at any time in the update process. It is easy to see theoretically that the mean fitness $f(t, 3)$ should always be $1/2$, no matter how many updates have been performed on the lattice. On the other hand, one can get from equation (6) a result for $f(t, 3) = f_{\mathrm{P}} = 0.47115$. The closeness of the result from the fitting ansatz with our exact theoretical expectation may indicate that equation (6) is, with quite high accuracy, true for all $L \geq 3$ and t and that other terms with higher powers in $1/L$ can be neglected. With equation (6) one can have

$$f(t, L) = f_\infty(t) - \frac{F(t)}{L} \ . \quad (7)$$

In the last equation $f_\infty(t) = (1 + G(t))/2$ is the mean fitness for an infinite lattice at scaled time t and can be related to the scaling solution $G(t)$ of the gap equation in the BS model for an infinite lattice, while $F(t)$ measures the finite-size effect. Using $f(t, 3) = 1/2$ one gets the finite-size correction from the last equation

$$F(t) = 3 \left(f_\infty(t) - \frac{1}{2} \right) \ . \quad (8)$$

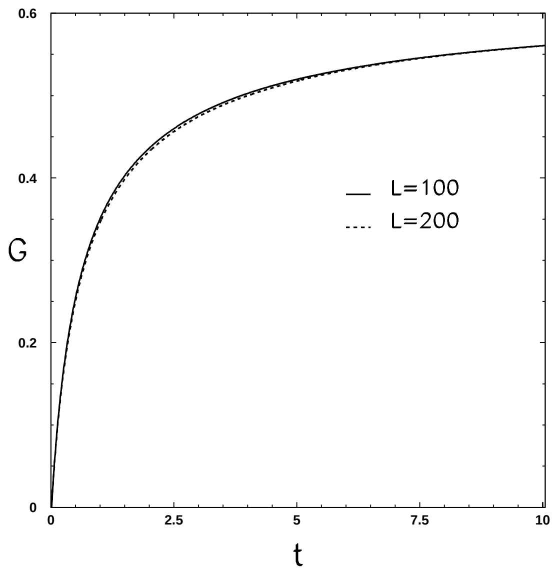

Fig. 4. Scaling solution $G(t)$ to the gap equation (1) for an infinite lattice. Two curves are obtained from simulations of $f(t, L)$ with lattice size $L = 100$ and 200, respectively.

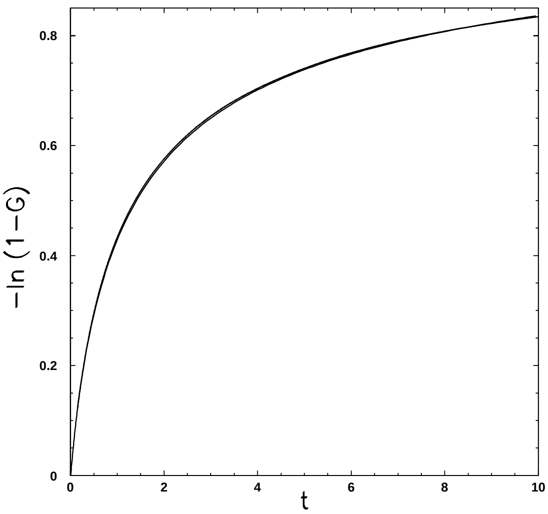

Fig. 5. $-\ln(1-G)$ as a function of t. The solid curve is from the curves in Figure 4 and the dashed curve from the parameterization equation (10).

Based on this ansatz one can extract $f_\infty(t)$ from simulations with finite lattice size L as

$$f_\infty(t) = \frac{1+G(t)}{2} = \frac{f(t,L) - 3/(2L)}{1 - 3/L}\,. \tag{9}$$

This gives us an opportunity to extract the limiting behavior of the mean fitness from a simulation with finite L. The obtained result for $f_\infty(t)$ from the simulation data with $L = 100$ is shown in Figure 3 as a thick solid curve. Collapsed curves can be obtained from simulation results with other L's. One can also try to get $f_\infty(t)$ by using equation (7) from two simulations with different L without using equation (8). The curve obtained in this way is also coincident with that shown in Figure 3. This means that higher order terms of $1/L$ in equation (7) can be neglected.

From these relations the scaling function $G(t)$ can be calculated from any set of data with finite L. We find that almost collapsed curves for $G(t)$ can be given from the six simulations we have performed. For illustration two calculated curves from simulations with $L = 100$ and 200 are shown in Figure 4 for t in the range (0, 10).

Because of the gap equation (1), it is more convenient to rewrite $-\ln(1-G)$ instead of G as a function of t, since the first derivative of $-\ln(1-G)$ with respect to t is just the reciprocal of $\langle S\rangle_G$. The behavior of $-\ln(1-G)$ is shown in Figure 5. 2000 data points extracted from the simulations are used in the figure. It is interesting to note here that the curve in Figure 5 can be very well parameterized by a simple expression

$$-\ln(1-G) = A\left(1 - \left(\frac{t_0}{t_0+t}\right)^{\delta} \frac{1+a_1 t + b_1 t^2}{1+a_2 t + b_2 t^2}\right), \tag{10}$$

with $A = 1.0996, \delta = 0.5883, t_0 = 0.965, a_1 = 2.2, a_2 = 2.5, b_1 = 0.24, b_2 = 0.21$. Though the parameters in the above expression can be calculated through complicated nonlinear fittings, they are, in fact, obtained in this paper from manual adjustment through comparing the curves from simulations and from the last expression. Since the curve from the expression almost coincides with the one from the simulations, parameters from nonlinear fitting should be very close to the ones given above. From this parameterization, one can find that $G \simeq A(\delta/t_0 + a_2 - a_1)t$ for $t \ll t_0$, and $f_c - G \propto t^{-\delta}$ for $t \to \infty$. The critical value f_c is determined from A through $f_c = 1 - \exp(-A) = 0.667$, and the exponent γ in the ansatz $\langle S\rangle_G \sim (f_c - G)^{-\gamma}$ is $(1+\delta)/\delta = 2.70$. The results for f_c and γ are in good agreement with those in [17] which were obtained from a different statistical method. This shows that the same information about the critical state can be obtained without the knowledge of the avalanche size distribution. The mean lifetime $\langle S\rangle_G$ of avalanches with gap G can be calculated from equations (10) and (1) and is shown in Figure 6 as a function of $f_c - G$. At first glimpse, it seems that $\langle S\rangle_G$ is a power of $f_c - G$ for the entire region shown. In fact, the slope decreases a little with the increase of $\ln(f_c - G)$. That means that $\langle S\rangle_G$ increases slowly with G for small G and quickly for larger G. Eventually $\langle S\rangle_G$ becomes divergent for $G \to f_c$. If one does a linear fit to the curve for a very small $\ln(f_c - G)$ region (namely, for G very close to its critical value), the exponent $\gamma = 2.70$ can be found.

4 Conclusions

In summary, a new method to study the self-organized criticality in the BS model is adopted in this paper. We suggest that a scaled time t would be a better variable for investigating the process approaching the critical state.

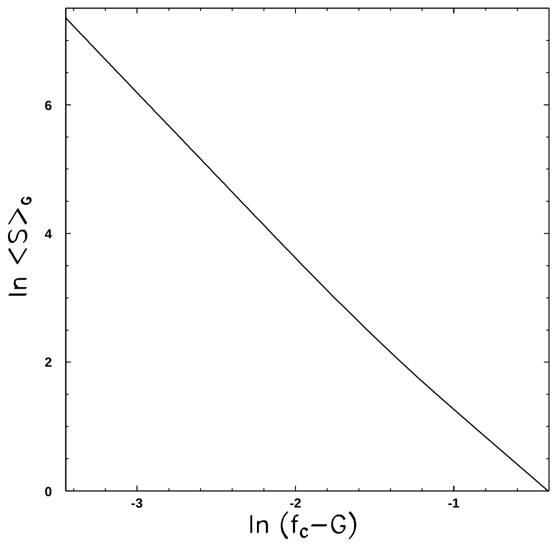

Fig. 6. $\ln\langle S\rangle_G$ as a function of $\ln(f_c - G)$ for an infinite lattice. $\langle S\rangle_G$ diverges with exponent $\gamma = 2.70$ when $G \rightarrow f_c = 0.667$.

The finite-lattice-size effect on the evolution is investigated from the fluctuations of the mean fitness f_i for the one-dimensional BS model in an event-by-event way, and a scaling solution for the gap equation is given for the first time, and a function $\langle S\rangle_G$ of G is also given numerically. We show that the critical value of the gap and the divergent exponent γ can be obtained from the numerical results.

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